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Cambridge International AS & A Level

MATHEMATICS (9709) P2

TOPIC WISE QUESTIONS + ANSWERS | COMPLETE SYLLABUS

PapaCambridge

Chapter 4

Differentiation

102. 9709_s20_qp_21 Q: 3

A curve has parametric equations

$$x = e^t - 2e^{-t}, \qquad y = 3e^{2t} + 1.$$

Find the equation of the tangent to the curve at the point for which $t = 0$. [5]

103. 9709_s20_qp_22 Q: 2

Find the exact coordinates of the stationary point on the curve with equation $y = 5xe^{\frac{1}{2}x}$. [5]

104. 9709_s20_qp_22 Q: 3

The equation of a curve is $\cos 3x + 5\sin y = 3$.

Find the gradient of the curve at the point $\left(\frac{1}{9}\pi, \frac{1}{6}\pi\right)$. [5]

105. 9709_w20_qp_21 Q: 7

A curve is defined by the parametric equations

$$x = 3t - 2\sin t, \qquad y = 5t + 4\cos t,$$

where $0 \leqslant t \leqslant 2\pi$. At each of the points P and Q on the curve, the gradient of the curve is $\frac{5}{2}$.

(a) Show that the values of t at P and Q satisfy the equation $10\cos t - 8\sin t = 5$. [3]

(b) Express $10\cos t - 8\sin t$ in the form $R\cos(t + \alpha)$, where $R > 0$ and $0 < \alpha < \frac{1}{2}\pi$. Give the exact value of R and the value of α correct to 3 significant figures. [3]

(c) Hence find the values of t at the points P and Q. [4]

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106. 9709_w20_qp_22 Q: 5

The equation of a curve is $2e^{2x}y - y^3 + 4 = 0$.

(a) Show that $\dfrac{dy}{dx} = \dfrac{4e^{2x}y}{3y^2 - 2e^{2x}}$. [4]

(b) The curve passes through the point (0, 2).

Find the equation of the tangent to the curve at this point, giving your answer in the form $ax + by + c = 0$. [3]

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(c) Show that the curve has no stationary points. [2]

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107. 9709_m19_qp_22 Q: 7

The parametric equations of a curve are

$$x = 2t - \sin 2t, \quad y = 5t + \cos 2t,$$

for $0 \leqslant t \leqslant \frac{1}{2}\pi$. At the point P on the curve, the gradient of the curve is 2.

(i) Show that the value of the parameter at P satisfies the equation $2\sin 2t - 4\cos 2t = 1$. [4]

(ii) By first expressing $2\sin 2t - 4\cos 2t$ in the form $R\sin(2t - \alpha)$, where $R > 0$ and $0 < \alpha < \frac{1}{2}\pi$, find the coordinates of P. Give each coordinate correct to 3 significant figures. [7]

108. 9709_s19_qp_21 Q: 3

Find the equation of the normal to the curve

$$x^2 \ln y + 2x + 5y = 11$$

at the point (3, 1). [7]

109. 9709_s19_qp_22 Q: 3

Find the exact coordinates of the stationary point of the curve with equation $y = \dfrac{3x}{\ln x}$. [5]

110. 9709_w19_qp_21 Q: 3

A curve has equation $y = \dfrac{3 + 2\ln x}{1 + \ln x}$. Find the exact gradient of the curve at the point for which $y = 4$. [5]

111. 9709_w19_qp_21 Q: 7

The equation of a curve is $x^2 - 4xy - 2y^2 = 1$.

(i) Find an expression for $\frac{dy}{dx}$ and show that the gradient of the curve at the point $(-1, 2)$ is $-\frac{5}{2}$. [5]

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(ii) Show that the curve has no stationary points. [3]

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(iii) Find the x-coordinate of each of the points on the curve at which the tangent is parallel to the y-axis. [2]

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112. 9709_w19_qp_22 Q: 5

Find the exact coordinates of the stationary point of the curve with equation $y = e^{-\frac{1}{2}x}(2x + 5)$. [5]

113. 9709_w19_qp_22 Q: 7

The parametric equations of a curve are

$$x = 3\sin 2\theta, \quad y = 1 + 2\tan 2\theta,$$

for $0 \leqslant \theta < \frac{1}{4}\pi$.

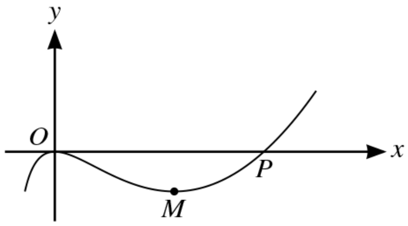
(i) Find the exact gradient of the curve at the point for which $\theta = \frac{1}{6}\pi$. [4]

(ii) Find the value of θ at the point where the gradient of the curve is 2, giving the value correct to 3 significant figures. [4]

114. 9709_m18_qp_22 Q: 2

A curve has equation $y = 4x \sin \frac{1}{2}x$. Find the equation of the tangent to the curve at the point for which $x = \pi$. [4]

115. 9709_m18_qp_22 Q: 7



The diagram shows part of the curve defined by the parametric equations

$$x = t^2 + 4t, \qquad y = t^3 - 3t^2.$$

The curve has a minimum point at M and crosses the x-axis at the point P.

(i) Find the gradient of the curve at P. [4]

(ii) Find the coordinates of the point M. [3]

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(iii) The value of the gradient of the curve at the point with parameter t is denoted by m. Show that

$$3t^2 - (2m + 6)t - 4m = 0$$

and hence find the set of possible values of m for points on the curve. [4]

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116. 9709_s18_qp_21 Q: 5

The parametric equations of a curve are

$$x = 2\cos 2\theta + 3\sin\theta, \qquad y = 3\cos\theta$$

for $0 < \theta < \frac{1}{2}\pi$.

(i) Find the gradient of the curve at the point for which $\theta = 1$ radian. [4]

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(ii) Find the value of $\sin\theta$ at the point on the curve where the tangent is parallel to the y-axis. [3]

117. 9709_s18_qp_22 Q: 2

A curve has equation $y = 3\ln(2x + 9) - 2\ln x$.

(i) Find the x-coordinate of the stationary point. [4]

(ii) Determine whether the stationary point is a maximum or minimum point. [2]

118. 9709_s18_qp_22 Q: 5

A curve has equation

$$y^3 \sin 2x + 4y = 8.$$

Find the equation of the tangent to the curve at the point where it crosses the y-axis. [6]

119. 9709_w18_qp_21 Q: 5

A curve has parametric equations

$$x = t + \ln(t + 1), \qquad y = 3te^{2t}.$$

(i) Find the equation of the tangent to the curve at the origin. [5]

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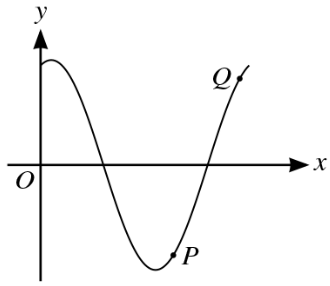
(ii) Find the coordinates of the stationary point, giving each coordinate correct to 2 decimal places. [4]

120. 9709_w18_qp_21 Q: 7



The diagram shows the curve with equation $y = \sin 2x + 3\cos 2x$ for $0 \leqslant x \leqslant \pi$. At the points P and Q on the curve, the gradient of the curve is 3.

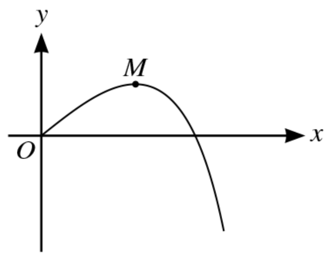
(i) Find an expression for $\frac{dy}{dx}$. [2]

(ii) By first expressing $\frac{dy}{dx}$ in the form $R\cos(2x + \alpha)$, where $R > 0$ and $0 < \alpha < \frac{1}{2}\pi$, find the x-coordinates of P and Q, giving your answers correct to 4 significant figures. [8]

Additional Page

If you use the following lined page to complete the answer(s) to any question(s), the question number(s) must be clearly shown.

121. 9709_w18_qp_22 Q: 3



The diagram shows the curve with equation

$$y = 5\sin 2x - 3\tan 2x$$

for values of x such that $0 \leqslant x < \frac{1}{4}\pi$. Find the x-coordinate of the stationary point M, giving your answer correct to 3 significant figures. [5]

122. 9709_w18_qp_22 Q: 4

Find the gradient of the curve

$$4x + 3ye^{2x} + y^2 = 10$$

at the point (0, 2). [5]

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123. 9709_m17_qp_22 Q: 4

Find the gradient of the curve

$$x^2 \sin y + \cos 3y = 4$$

at the point $(2, \frac{1}{2}\pi)$. [6]

124. 9709_s17_qp_21 Q: 7

The parametric equations of a curve are

$$x = t^3 + 6t + 1, \qquad y = t^4 - 2t^3 + 4t^2 - 12t + 5.$$

(i) Find $\frac{dy}{dx}$ and use division to show that $\frac{dy}{dx}$ can be written in the form $at + b$, where a and b are constants to be found. [5]

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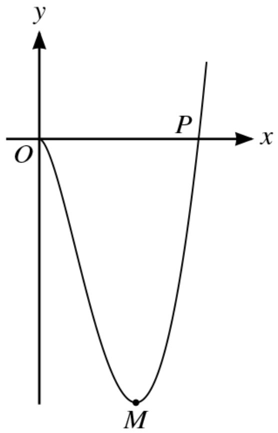
(ii) The straight line $x - 2y + 9 = 0$ is the normal to the curve at the point P. Find the coordinates of P. [3]

125. 9709_s17_qp_21 Q: 8



The diagram shows the curve with equation

$$y = 3x^2 \ln(\tfrac{1}{6}x).$$

The curve crosses the x-axis at the point P and has a minimum point M.

(i) Find the gradient of the curve at the point P. [5]

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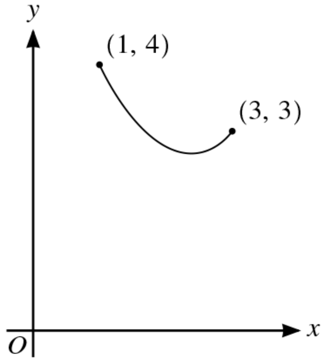
(ii) Find the exact coordinates of the point M. [5]

126. 9709_s17_qp_22 Q: 4

Find the equation of the tangent to the curve $y = \dfrac{e^{4x}}{2x+3}$ at the point on the curve for which $x = 0$. Give your answer in the form $ax + by + c = 0$ where a, b and c are integers. [5]

127. 9709_s17_qp_22 Q: 8



The diagram shows the curve with parametric equations

$$x = 2 - \cos 2t, \qquad y = 2\sin^3 t + 3\cos^3 t + 1$$

for $0 \leqslant t \leqslant \frac{1}{2}\pi$. The end-points of the curve are (1, 4) and (3, 3).

(i) Show that $\dfrac{dy}{dx} = \frac{3}{2}\sin t - \frac{9}{4}\cos t$. [5]

(ii) Find the coordinates of the minimum point, giving each coordinate correct to 3 significant figures. [3]

(iii) Find the exact gradient of the normal to the curve at the point for which $x = 2$. [3]

128. 9709_w17_qp_21 Q: 6

The parametric equations of a curve are

$$x = 2e^{2t} + 4e^{t}, \qquad y = 5te^{2t}.$$

(i) Find $\frac{dy}{dx}$ in terms of t and hence find the coordinates of the stationary point, giving each coordinate correct to 2 decimal places. [6]

(ii) Find the gradient of the normal to the curve at the point where the curve crosses the x-axis. [3]

129. 9709_w17_qp_22 Q: 3

The equation of a curve is $y = \tan \frac{1}{2}x + 3 \sin \frac{1}{2}x$. The curve has a stationary point M in the interval $\pi < x < 2\pi$. Find the coordinates of M, giving each coordinate correct to 3 significant figures. [6]

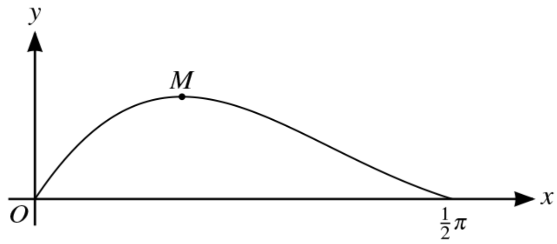
130. 9709_w17_qp_22 Q: 7

The equation of a curve is $x^2 + 4xy + 2y^2 = 7$.

(i) Find the equation of the tangent to the curve at the point $(-1, 3)$. Give your answer in the form $ax + by + c = 0$ where a, b and c are integers. [6]

(ii) Show that there is no point on the curve at which the gradient is $\frac{1}{2}$. [4]

131. 9709_m16_qp_22 Q: 6



The diagram shows the part of the curve $y = 3e^{-x} \sin 2x$ for $0 \leqslant x \leqslant \frac{1}{2}\pi$, and the stationary point M.

(i) Find the equation of the tangent to the curve at the origin. [4]

(ii) Find the coordinates of M, giving each coordinate correct to 3 decimal places. [4]

132. 9709_m16_qp_22 Q: 7

The equation of a curve is $2x^3 + y^3 = 24$.

(i) Express $\frac{dy}{dx}$ in terms of x and y, and show that the gradient of the curve is never positive. [4]

(ii) Find the coordinates of the two points on the curve at which the gradient is -2. [5]

133. 9709_s16_qp_21 Q: 1

Find the gradient of the curve

$$y = 3e^{4x} - 6\ln(2x + 3)$$

at the point for which $x = 0$. [3]

134. 9709_s16_qp_21 Q: 5

A curve is defined by the parametric equations

$$x = 2\tan\theta, \quad y = 3\sin 2\theta,$$

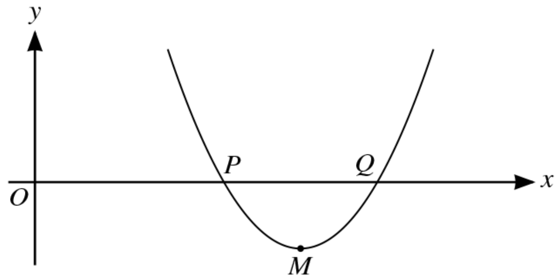
for $0 \leqslant \theta < \frac{1}{2}\pi$.

(i) Show that $\frac{dy}{dx} = 6\cos^4\theta - 3\cos^2\theta$. [4]

(ii) Find the coordinates of the stationary point. [3]

(iii) Find the gradient of the curve at the point $(2\sqrt{3}, \frac{3}{2}\sqrt{3})$. [2]

135. 9709_s16_qp_22 Q: 7



The diagram shows the curve with parametric equations

$$x = 2 - \cos t, \quad y = 1 + 3\cos 2t,$$

for $0 < t < \pi$. The minimum point is M and the curve crosses the x-axis at points P and Q.

(i) Show that $\frac{dy}{dx} = -12\cos t$. [4]

(ii) Find the coordinates of M. [2]

(iii) Find the gradient of the curve at P and at Q. [4]

136. 9709_w16_qp_21 Q: 3

A curve has equation $y = 2\sin 2x - 5\cos 2x + 6$ and is defined for $0 \leqslant x \leqslant \pi$. Find the x-coordinates of the stationary points of the curve, giving your answers correct to 3 significant figures. [6]

137. 9709_w16_qp_21 Q: 6

The equation of a curve is $3x^2 + 4xy + y^2 = 24$. Find the equation of the normal to the curve at the point (1, 3), giving your answer in the form $ax + by + c = 0$ where a, b and c are integers. [8]

138. 9709_s15_qp_21 Q: 3

The equation of a curve is

$$y = 6\sin x - 2\cos 2x.$$

Find the equation of the tangent to the curve at the point $\left(\frac{1}{6}\pi, 2\right)$. Give the answer in the form $y = mx + c$, where the values of m and c are correct to 3 significant figures. [5]

139. 9709_s15_qp_21 Q: 7

The equation of a curve is

$$y^3 + 4xy = 16.$$

(i) Show that $\dfrac{dy}{dx} = -\dfrac{4y}{3y^2 + 4x}$. [4]

(ii) Show that the curve has no stationary points. [2]

(iii) Find the coordinates of the point on the curve where the tangent is parallel to the y-axis. [4]

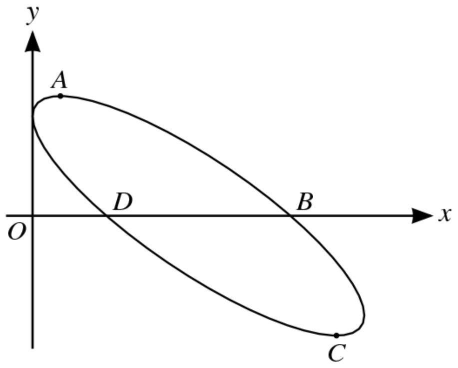
140. 9709_w15_qp_21 Q: 2

A curve has equation

$$y = \frac{3x + 1}{x - 5}.$$

Find the coordinates of the points on the curve at which the gradient is -4. [5]

141. 9709_w15_qp_21 Q: 7



The parametric equations of a curve are

$$x = 6\sin^2 t, \qquad y = 2\sin 2t + 3\cos 2t,$$

for $0 \leqslant t < \pi$. The curve crosses the x-axis at points B and D and the stationary points are A and C, as shown in the diagram.

(i) Show that $\frac{dy}{dx} = \frac{2}{3}\cot 2t - 1$. [5]

(ii) Find the values of t at A and C, giving each answer correct to 3 decimal places. [3]

(iii) Find the value of the gradient of the curve at B. [3]

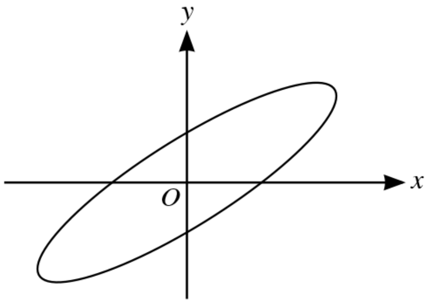
142. 9709_w15_qp_22 Q: 5

Find the x-coordinates of the stationary points of the following curves:

(i) $y = 4xe^{-3x}$; [3]

(ii) $y = \dfrac{4x^2}{x+1}$. [5]

143. 9709_w15_qp_22 Q: 6



The diagram shows the curve with parametric equations

$$x = 3\cos t, \qquad y = 2\cos(t - \tfrac{1}{6}\pi),$$

for $0 \leqslant t < 2\pi$.

(i) Show that $\dfrac{dy}{dx} = \frac{1}{3}(\sqrt{3} - \cot t)$. [5]

(ii) Find the equation of the tangent to the curve at the point where the curve crosses the positive y-axis. Give the answer in the form $y = mx + c$. [4]

144. 9709_w15_qp_23 Q: 3

The parametric equations of a curve are

$$x = (t+1)e^t, \qquad y = 6(t+4)^{\frac{1}{2}}.$$

Find the equation of the tangent to the curve when $t = 0$, giving the answer in the form $ax + by + c = 0$ where a, b and c are integers. [6]

145. 9709_w15_qp_23 Q: 7

The equation of a curve is $y = \dfrac{\sin 2x}{\cos x + 1}$.

(i) Show that $\dfrac{dy}{dx} = \dfrac{2(\cos^2 x + \cos x - 1)}{\cos x + 1}$. [7]

(ii) Find the x-coordinate of each stationary point of the curve in the interval $-\pi < x < \pi$. Give each answer correct to 3 significant figures. [3]